Managerial incentives from option compensation and risky guarantees

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Abstract

This paper solves the dynamic investment problem of a risk averse agent compensated with a performance related bonus plus a salary guaranteed up to a certain level of underperformance. The main contribution is to explicitly take into account the financial fragility of the principal [employer], by making the tolerated underperformance a function of the principal's net asset value.

The model predicts that an agent who underperforms reverts much quicker to her benchmark when her principal experiences financial stress.

Our model, combined with the fact that the 2008 crisis shook many financial corporations, permits to explain the observation of Petajisto (2013) that 'closet indexing increases in volatile and bear markets and has become more popular after 2007' (p1) and the more global declining active risk taking in investment funds. By contrast, previous structural models such as Koijen (2014) or Buraschi et al. (2014), because they do not incorporate the principal's financial fragility, cannot describe time variations in risk-taking beyond the business cycle.

Empirical, the managers' unconditional risk-taking behaviour (without taking her relative performance into account) is more influenced by firm-level conditions than by other factors, and we expect to show that the main driver is precisely the financial fragility of the principal.

Keywords: Mutual fund dynamic risk-taking, Structural estimation, Dynamic Asset Allocation, Risk-Shifting, Option compensation, Managerial Incentives.

JEL: G11, G23, G31, G32, C61.

1 Introduction

The asset management literature devotes a large attention to the impact of incentives on manager's risk taking behaviour, which crucially impacts large areas of the financial industry. The same mix of risky financial guarantees and option-based compensation being at stake in levered financial institutions as a whole and in the branches of financial conglomerates, the effects of such incentives are crucial for regulators as well as for those who design intra-group guarantees and financial support.

In delegated portfolio management, a structural model is needed to explain state-dependent risk taking (Koijen, 2014).[1] The main contribution of this paper is to provide a structural model for manager's risk-taking that takes into account the fragility of the principal who provides guarantees.

The manager is endowed with (1) a performance-related bonus, (2) a fixed salary payable up to a certain level of underperformance, where (3) the tolerated underperformance is an increasing function in the principal's wealth [net asset value].

Conditional on a given [fixed] value of the principal's wealth, (1) and (2) provide risk shifting and leverage incentives when the fund has a modest relative underperformance.

More precisely, (1) and (2) constitutes a non-linear remuneration contract which yields non-linear, state dependent allocation to a speculative portfolio defined, depending on the context, as a mix of the mean-variance efficient [MVE] portfolio and of an orthogonal portfolio which purely reflects the specific skills of the manager. The allocation to the speculative portfolio is humped shape between a minimum acceptable underperformance and a high reference point, endogeneously determined, which has the interpretation of a high-water mark.

To analyse (3), consider that a fall in the principal's wealth diminishes the tolerated underperformance and risk-shifting incentives, thus diminishing the value of the manager's remuneration contract; then, because the principal's wealth is stochastic, the manager hedges

[1] This means that linear performance regressions would be biased and inefficient to measure the manager's alpha or skills. With such incentives, the relative performance of delegated assets is a non-linear function of the manager's speculative portfolio – the efficient mean-variance portfolio if the manager has no specific skills.

this change by being under-invested in the assets that correlate with the principal's wealth, typically its shares.

We show the empirical validity of this model by showing that managers respond to the variation in their principal's health [their employer, the management company]. The analysis is preliminary, but, interestingly, when analysing Petajisto (2013)'s data, we find that the variation in risk-taking from peer managers (obtained by the author after matching this database with the fundamental information from CRSP) is the primary source of variation in manager's *active share* and *tracking error*.

Fuzzy matching between the name of the asset management company and a database for financial institutions will permit a more accurate quantification of the impact of the principal's financial fragility on the agent's behaviour – the share price or credit spreads of the principal can serve as proxies for its financial fragility, for the subset of companies that are listed.

Related Literature

Our analysis of the dynamic investment problem of a manager with option compensation but risky guarantees relates to Carpenter (2000), who studies the risk-shifting behaviour from an asset manager with a guaranteed income plus a performance-related bonus. It also relates to (Berkelaar and Post, 2004), who detail the investment strategy of a loss-averse investor, and to Basak and Shapiro (2001), who study an investor with Value-at-Risk constraints, because in all cases the utility function of the agent is (locally) non-concave.

More recently, Buraschi et al. (2014) extended the model of Koijen (2014) to study hedge fund incentives with incentives to limit underperformance to a floor (which can be interpreted in our context by a limited external guarantee from the principal).

We extend these problems by considering that the floor, or guarantee offered to the agent, is stochastic and depends on the principal's wealth.

The paper is organised as follows:

The following subsection 2 defines the economy, compensation contract and optimisation problem, section 3 studies the optimal risk-shifting program. 4 gives a detailed analysis of a simple case, with 4.1 focussing on numerical integration and 4.2 deriving the weight strategy. Section 5 summarises the preliminary results from the on-going empirical analysis, and 6 concludes.

Appendices are organised as follows: A defines notations more accurately, B derives the optimal program, C provides a numerical and graphical illustration, and D provides a generalisation of the simple model to various settings (numeraires, definition of manager's remuneration, utility function), and E details the link between this remuneration contract and those studied earlier.

2 Summary of Notations and Problems Studied

2.1 The economy

We live in a continuous-time Black-Scholes economy where the benchmark L has volatility σ_L.

As formalised by El Karoui et al. (2005), the optimal portfolio in the presence of constraints on the utility function is an option on an optimal 'unconstrained' portfolio. In the case of utility on terminal wealth, the unconstrained portfolio has the form $A^u_T = (v_0 M_T)^{(-1/\gamma)}$ where M is the stochastic discount factor ($dM_t = M_t \cdot (-r \cdot dt - \theta \cdot dW)$). If the utility is CRRA on the ratio X_T/L_T, the unconstrained portfolio is (proportional to) $A^u_T = (L_T)^{(1-1/\gamma)} \cdot (M_T)^{(-1/\gamma)}$, aggregating two risk drivers.[2] If as in Koijen (2014) or Buraschi et al. (2014), the manager has the skilled to generate above market returns, A^u will be a speculative portfolio which weights π^u will combine the MVE and an orthogonal long-short portfolio which reflects her extra skills.

We will thus achieve notational economy and general notations by using A^u as the driver of the strategy. F^u and G have correlation ρ.

m^* is multiplicative constant that scales the 'participation' of optimal, option-like strategies to the underlying unconstrained A^u.

[2] In the former case, A^u has volatility $\sigma_{A^u} = \frac{||\theta||}{\gamma}$, in the later A/L has volatility $\sigma_{F^u} = \frac{||\theta - \sigma_L||}{\gamma}$

See Appendix A for details on the economy and notations.

The agent can design her strategy to extract wealth from the principal. The solution will be noted $A_T^* = up_T + put_T$.

2.2 The manager's remuneration

A manager receives a fixed wage compensation which in most relevant situations entails some risk, for instance job loss if the principal's financial situation deteriorates strongly while the manager under-performs, an aspect which can be modelled as a default risk of the principal when the agent's losses are high compared to the principal's wealth.

So, the agent we study has *limited liability* up to a certain under-performance that depends on the stochastic wealth of the principal.[3] The manager's compensation reads:

$$X_T = \alpha \cdot (A_T - L_T)^+ + K \cdot L_T \cdot \mathbb{1}_{\beta \cdot [L_T - A_T]^+ \leq m_G \cdot Y_T} \tag{1}$$

where Y_T is the principal's net asset value, and $m_G \leq 1$ is the fraction of the net asset value that the principal is ready to allocate to the agent to compensate for the losses that result from underperformance $\beta \cdot [L_T - A_T]^+$, and α is the number of options granted to the manager.[4]

2.3 The manager's utility function

The manager has a simple CRRA utility with risk aversion γ.

[3] It could be chosen to model the financial fragility of a third party*e.g.,* that of the prime broker. This represents the case when G, the net asset value of the provider of the guarantee, is directly traded.

[4] Generally, the principal's tolerance for under-performance by the agent is not only limited, it also depends on the residual net asset value of the principal. A financial conglomerate, even provides a formal guarantee to each of its subsidiaries, may limit it to a fraction of its net wealth, so $m_G << 1$ but in the order of $10\% - 30\%$ and $\beta = 1$ is to be expected. In fund management, if there are no performance fees, it is expected that $\beta < \alpha$ and these be in the range of $.05\% - 5\%$. If there are performance fees, then while β remains in the same range, α, the fraction of the performance given to the manager, could be in a $1\% - 15\%$ range. Note that in the case of high water mark, the initial value of the benchmark could be above one, or $L_0 > A_0$ is possible. The principal of a defined-benefit pension plan makes good on the plan deficit, up to a value bounded by its net asset value, and in this very specific case one may consider $m_G = 1$ with $\beta = 1$, which means that the guarantee cannot be legally limited.

$$U(X_T) = \frac{(X_T/L_T)^{(1-\gamma)}}{1-\gamma} \tag{2}$$

The use of a common numeraire L means that is is efficient to use ratios of asset values to liabilities as notional quantities. We denote $G = \frac{Y}{L}$ and $F = \frac{A}{L}$ (these quantities are martingales under the forward-measure $\mathbb{Q}_L$).

2.4 Main insight

In this paper we study how the agent optimally behaves when she aims to simultaneously secure the promise made to him and optimally exploit the remuneration incentives given to him by the option-based compensation.

Conditionally to a given G_T, it is easy to see that the problem is similar either to that of Buraschi et al. (2014) (this happens when $m_G \cdot G_T < k$) or to that of Carpenter (2000), namely when $m_G \cdot G_T > k$.

The stochastically limited guarantee introduces new features to the optimal asset allocation by the agent. The risk in going from the Carpenter (2000) model to the Buraschi et al. (2014) model is hedged, *i.e.*, the agent also has a hedging demand to offset the impact of a deterioration in principal's wealth [risk tolerance].

In addition, a reduction in the principal's net wealth reduces its tolerance for under-performance, thus reducing the agent's leverage.

The dependence of the agent on the principal's wealth in expected to lead fund managers to increase their tracking error to make up for individual under-performance when the management company experiences profits, while during times of market crash, when the management company experiences losses, fund managers are expected to act much more prudently and revert to their benchmarks.

Although this setting is very simple, the non-convex region is identical over a variety of configurations, and various extensions are given in Appendix D.

3 Maximum Risk Shifting

3.1 Incentives

In such agency problem, the agent is generally given an option or participation to the success of the firm to give him incentives to exert effort. However, the guaranteed income in case of failure gives incentives to gambling. One can think of a father and son relationship, where the son may enter a risky career because he knows he will benefit from the unconditional support of his father in case of failure.

In the static optimisation problem at time T, for any given G_T, the (conditional) non-convex optimisation problem is similar to the one in Carpenter (2000), Berkelaar and Post (2004), or more recently Buraschi et al. (2014) and illustrated in figure 1.

In our case, the agent retrieves the same utility for any limited shortfall compared to her benchmark ($\beta[A_T^* - L_T] \leq m_G \cdot Y_T$) that the utility of $k \cdot L_T$. As it Carpenter (2000), it can be stated trivially that A^* will never take any value in the open interval $]A_T^-, L_T[$ where $A^-(Y_T, L_T) = [L_T - \frac{m_G}{\beta} Y_T]^+$, so equivalently F^* will never take any value in $]F^-(G_T), 1[$, where $F^-(G_T) = [1 - \frac{m_G}{\beta} G_T]^+$

Thus, in this problem with an external guarantee, the agent extracts as much wealth as she can from the principal. The guarantee also induces non-convexity, and the manager will only target an asset value at least equal to the tangency of the utility to A^-, conditional on these two values.

In Carpenter (2000), $A^- = 0$ independently of L_T (or any other factor), so that A^+ also is fixed, and the analysis can be pursued according to A^+ defined such as $\frac{U(A^+) - U(A^-)}{A^+ - A^-} = dU(A^+)$ where $U(A)$ is a shortcut notation for $U(X(A))$, *i.e.*, $U(A^-) = U(K)$.

Figure 1: Visual interpretation of the strategy

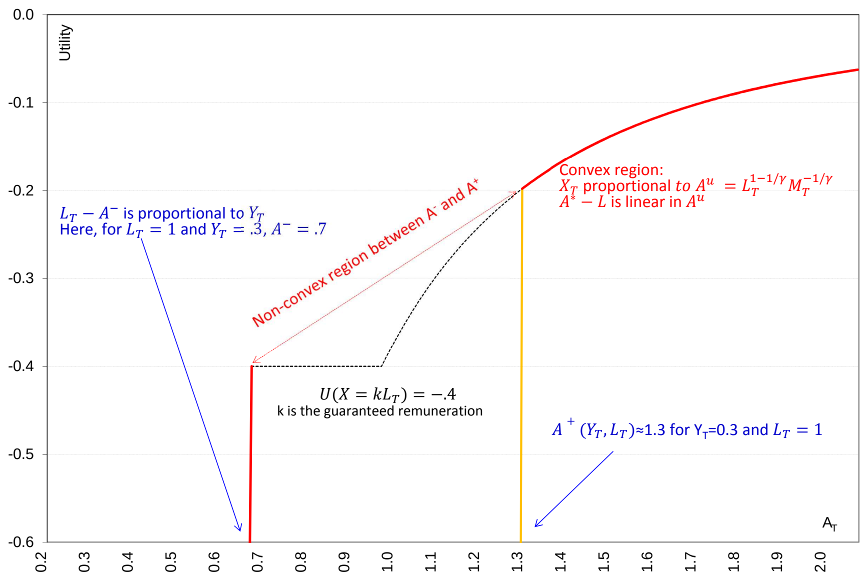


Figure 1 illustrates the non-convexity that arises conditional to a given value of Y_T and L_T, for varying levels of A_T^u. Here, $\frac{m_G}{\beta} Y_T = 0.3$ (denoted m_G in the graph) and $L_T = 1$ (alternatively, $m_G \cdot G_T = .3$ and the x-axis represents F_T^u). In case of success, the agent has total wealth greater than $X^+(.3) \approx 1.3$. The utility of the terminal A_T—before the principal makes good on losses—is non-convex: the utility of any $A_T < L_T$ is that of $[k - G_T]^+ = .7$. Between $k = 1$ and $A^+(G_T) \approx 1.3$, the agent also prefers in expectation reaching $A^+(G_T) \approx 1.3$ with some probability and $A_-(G_T) \approx .7$ with another. $A_-(G_T)$ is stochastic and is lower when the principal is richer because larger recovery contributions are then possible, and the reverse is true for $A^+(G_T)$.

The non-contractible agent follows the following program:

The solution will be noted $F_T^* = up_T + put_T$, where up_T delivers $(1 + \frac{1}{\alpha}(m \cdot F_T^u - k)$ in the exercise region $m^* \cdot F_T^u > F^+(G_T) \geq k$, while $put_T = [1 - \frac{m_G}{\beta} G_T]^+$ when up_T is not exercised.

Theorem 3.1. *Maximum Risk Shifting Program*

$$F_T^* = [1 - \frac{m_G}{\beta} G_T]^+ \cdot 1_{m^* \cdot F_T^u < F^+(G_T)} + (1 + (m \cdot F_T^u - k)/\alpha) \cdot 1_{m^* \cdot F_T^u \geqslant F^+(G_T)} \tag{3}$$

where the line $((F^-, U(K)), (F^+, U(F^+)))$ *is tangential to the utility curve at* $(F^+, U(F^+))$.

The agent achieves a partly binary payoff, ending either with significant profits or with losses at the maximum level that can be tolerated by her principal. Risk-shifting and risk-management coexist, because a minimum funding value needs to be secured. Furthermore, all values (and thus the replicating strategy) will depend on G_T.

However, because the risk in the guarantee is driven by G, the stochastic G needs to be taken into account in the dynamic replicating strategy (see weight strategy).

3.1.1 Implementation with the Principal's Shares

In our Black-Scholes economy, ρ_{GF} is a constant, or would be a deterministic function of t if interest-rates are a Vasicek.[5]

Share Value

The terminal payoffs at time T^+ to the principal / shareholders (after losses from maintaining the salary are discounted from the principal's NAV) are:

$$S_T = \begin{cases} G_T & \text{if} \quad m^* \cdot F_T^u > F^+(G_T) \\ G_T - k & \text{if} \quad m_G \cdot G_T > k \text{ and } m^* \cdot F_T^u \leqslant F^+(k) \\ (1-m_G) \cdot G_T & \text{if} \quad m_G \cdot G_T < k \text{ and } m^* \cdot F_T^u \leqslant F^+(G_T) \end{cases} \tag{4}$$

When $m_G \cdot G_T < k$, the principal only commits the faction m_G of his wealth to support the agent, the shareholder receiving $1 - m_G$ (in case of underperformance). Then $[\frac{\partial S_T}{\partial G_T} | m_G \cdot G_T < k \ , \ m^* \cdot F_T^u \leqslant F^+(G_T)] = (1 - m_G)$ and principal shares S are hedging instruments for G. Because only put_T has value in the exercise region ($up_T = 0$),

$$\frac{\partial F_t^*}{\partial S_t} |[m^* \cdot F_t^u < k \ , \ m_G \cdot G_t < k] \xrightarrow[t \to T]{} -\frac{m_G}{1 - m_G}$$

When one trades principal shares, the number of principal's shares traded by the agent in order to hedge principal risk reads $\frac{\partial F_t^*}{\partial S} = \frac{\partial F_t^*}{\partial G} / \frac{\partial S}{\partial G}$. However, S thus also is sensitive to F^u (via the probability of success), and thus to the performance of the MVE portfolio. So, in general, one must adjust the position in the underlying F_t^u as follows: $\frac{\partial F^*}{\partial F^u} - \frac{\partial F^*}{\partial S} \cdot \frac{\partial S}{\partial F^u}$

Remark 3.2. Note that the agent can hedge her exposure to the principal's net asset value G by dynamically trading S, up to the point where the positions can effectively be traded. When $m_G = 1$, the shares become insensitive to G in the region where $m_G < k$, at which point they cannot be relied upon for hedging changes in G.

[5] With a CRRA utility on wealth, this means that $\rho_{Y,MVE}$ the correlation between the principal's wealth and the MVE is constant, or a deterministic function of time.

4 Price and weight strategy

4.1 Pricing via numerical integration

Notations and generic tools

We denote $\mathbb{X}(\cdots) = \mathbb{E}^{\mathbb{Q}_X}[\mathbb{1}_{(\cdots)}]$ with X either L, Y or A^u.[6] The events described in $(\cdots)$ are of the form $(<> F^+, <> k)$. Here, $(> F^+, < k)$ indicate the events $(mF_T^u \geqslant F^+(m_G G_T)$, $m_G G_T < k)$, while $(<,<)$ indicate $(mF_T^u < F^+(m_G G_T)$, $m_G G_T < k)$, and $(\cdot)$ leaves a variable unspecified: $(> F^+) \equiv (> F^+, \cdot)$ indicates the events $(mF_T^u \geqslant F^+(G_T)\,,\ \forall G_T)$.

Then:

$$\mathbb{X}(> F^+, < k) = \mathbb{E}^{\mathbb{Q}_X}[\mathbb{1}_{m \cdot F_T^u \geqslant F^+(G_T), m_G G_T < k}] \tag{5}$$

$$\mathbb{X}(<,<) = \mathbb{E}^{\mathbb{Q}_X}[\mathbb{1}_{m \cdot F_T^u < F^+(G_T), m_G G_T < k}] \tag{6}$$

In general terms, we note that the numerical computation of these integrals is as follows:

$$\mathbb{X}(> F^+, <) = \int_{-\infty}^{-d_X(m_G G_t, k)} n(v) \cdot N\left(\frac{d_X(mF_t^u, F^+(v)) + \rho \cdot v}{\sqrt{1-\rho^2}}\right) \cdot dv$$

$$\mathbb{X}(<,<) = \int_{-\infty}^{-d_X(m_G G_t, k)} n(v) \cdot N\left(-\frac{d_X(mF_t^u, F^+(v)) + \rho \cdot v}{\sqrt{1-\rho^2}}\right) \cdot dv$$

where the function $d_X(S_t, K)$ denotes the traditional Black-Scholes normalised moneyness (position of the strike K in the forward distribution of S_T given S_t, where S is either G or F^u),[7] under the probability measure where X $(P, F^u$, or $G)$ is the numeraire.

$$\begin{aligned} d_L(S_t, K) =& \frac{ln(S_t/K) - 1/2 \cdot \sigma_S^2 \cdot (T-t)}{\sigma_S \cdot \sqrt{T-t}} \\ \text{For } X \neq L, \quad d_X(S_t, K) =& d_L(S_t, K) + \rho_{SX} \sigma_X \sqrt{T-t} \end{aligned} \tag{7}$$

where integration is performed across $v = d_X(G_t, G_T)$, the standardised values of G_T, and where $F^+(v)$ is a shorthand notation for $F^+(G^X(v))$, with $G^X(v)$ the inverse function of d_X

[6] Y and Y are used for the *asset-forward numeraire* as in any normal pricing formula, and under $\mathbb{Q}_L$ the ratio $F_t = A_t/L_t$ is a Martingale.

[7] The parameters σ_S, σ_X, ρ_{SX}, t and T are implied.

relative to G_T.[8]

Assuming $\beta > k$,[9] we have $F^+(m_G G_T | m_G G_T > k) = F^+(k)$. Then $\mathbb{X}(\cdot, > k)$ is a Normal *c.d.f.*, $\mathbb{X}(<, > k)$ and $\mathbb{X}(>, > k)$ are Bivariate *c.d.f.*, while $\mathbb{X}(\cdot, <)$, $\mathbb{X}(> F^+, <)$ and $\mathbb{X}(< F^+, <)$ require numerical integration. for instance, with the above notations,

$$\mathbb{A}(>, >) = \mathbb{E}_t^{\mathbb{Q}_A}[\mathbb{1}_{m \cdot F_T^u \geqslant F^+(k), m_G G_T > k}] = B(d_F(m_G G_t, k), d_F(m F_t^u, F^+(k))$$

Then it will be handy to compute separately $\mathbb{X}(> F^+, \cdot) = \mathbb{X}(> F^+, >) + \mathbb{X}(> F^+, <)$. for instance, $\mathbb{A}(> F^+) = \underbrace{\mathbb{A}(> F^+, < k)}_{(8)} + \underbrace{\mathbb{A}(>, >)}_{(9)}$

$$\mathbb{A}(> F^+) = \underbrace{\mathbb{A}(> F^+, <)}_{(8)} + \underbrace{\mathbb{A}(> F^+, <)}_{(9)}$$

$$\mathbb{A}(> F^+) = \left[\underbrace{\mathbb{E}_t^{\mathbb{Q}_A}[\mathbb{1}_{m \cdot F_T^u \geqslant F^+(k), m_G G_T > k}]}_{(8)} + \underbrace{\mathbb{E}_t^{\mathbb{Q}_A}[\mathbb{1}_{m \cdot F_T^u \geqslant F^+(G_T), m_G G_T < k}]}_{(9)} \right]$$

We also note that for any smooth concave utility, $F^+(G_T)$ is a differentiable function, and $d_X(F^+(v)$, $\mathbb{X}()$, up_t and put_t have continuous derivatives *w.r.t.* F_t^u and G_t.

Then, with a CRRA utility, the *put* and *up* options are integrals of a stream of Black-Scholes *univariate* options, which ensures that this strategy can be replicated if F^u and G are traded, and the derivation of the weight strategy is detailed in Section 4.2 on the facing page.

Pricing

From (3), we have $F_T^* = up_T + put_T$, with:

$up_T = 1 + \frac{1}{\alpha}(m \cdot F_T^u - k) \cdot \mathbb{1}_{(> F^+, \cdot)}$,

and $put_T = [1 - \frac{m_G}{\beta} G_T]^+ \cdot \mathbb{1}_{(< F^+, \cdot)}$

So $up_t = \frac{m}{\alpha} \cdot F_t^u \mathbb{E}_t^{\mathbb{Q}_A}[1_{(> F^+, \cdot)}] + (1 - k/\alpha) \cdot \mathbb{E}_t^{\mathbb{Q}_L}[1_{(> F^+, \cdot)}]$ can be written as $up_t = \frac{m}{\alpha} \cdot F_t^u \mathbb{A}(> F^+, \cdot) + (1 - k/\alpha) \cdot \mathbb{L}(> F^+, \cdot)$.

While $put_T = [1 - \frac{m_G}{\beta} G_T]^+ \cdot \mathbb{1}_{(< F^+, \cdot)} = [1 - \frac{m_G}{\beta} G_T]^+ \cdot \mathbb{1}_{(< F^+, <)}$ since in the region $(< F^+, > k)$ the manager achieves $A_T^* \equiv 0$ as in Carpenter (2000). Thus for pricing,

$put_t = \mathbb{L}(<, <) - \frac{m_G}{\beta} \cdot G_t \cdot \mathbb{Y}(<, <)$

Remark 4.1. In fact, $(\cdot, > k)$ is a region where the principal provides virtually unlimited

[8]Formally speaking, we use the inverse function of $d_X(S_t, K)$ relative to the ('variable') 'strike' K, but we now normalise all possible values of the underlying, not just one strike.

[9]When $\beta < k$, the threshold for $m_G \cdot G_T$ is β instead of k. Said otherwise, $F^- = 0$ for $m_G \cdot G_T = min(\beta, k)$).

support (is willing to accept any under-performance) to the agent, with the value of the funds falling all the way to zero in case her objectives are not met. In practice this may not be a desirable feature, so the principal could design more explicit contractual or incentives floor to underperformance.

4.2 Delta-Hedging

Semi-closed form pricing formulaes are derived for the current problem, and the level of complexity remains minimal for this problem thanks to the constant opportunity set and the remuneration is proportional to the benchmark. Other cases require an additional level of integration (usually without change in the interpretation).

Deltas of $B(\dots)$ Recalling the notation on page 11: $\mathbb{X}(>F^+,>) = \mathbb{E}^{\mathbb{Q}_X}[\mathbb{1}_{m\cdot F_T^u \geqslant F^+(G_T), m_G\cdot G_T > k}]$, for $\mathbb{X} = (\mathbb{L}, \mathbb{A}, \mathbb{Y})$, we have:

$$\frac{\partial \mathbb{X}(>,>)}{\partial F_t^u} = N\left(\frac{d_X(m_G G_t, k) - \rho d_X(mF_t^u, F^+(k))}{\sqrt{1-\rho^2}}\right)\cdot \frac{n(d_X(mF_t^u, F^+(k))}{\sigma_F\cdot\sqrt{T-t}\cdot\sqrt{1-\rho^2}}$$

$$\frac{\partial \mathbb{X}(>,>)}{\partial G_t} = N\left(\frac{d_X(mF_t^u, F^+(k)) - \rho d_X(G_t, k)}{\sqrt{1-\rho^2}}\right)\cdot \frac{n(d_X(m_G G_t, k))}{\sigma_G\cdot\sqrt{T-t}\cdot\sqrt{1-\rho^2}}$$

deltas of $\mathbb{X}(>F^+,<)$

$$\frac{\partial \mathbb{X}(>F^+,<)}{\partial F_t^u} = \int_{-\infty}^{-d_X(m_G G_t, k)} n\left(\frac{d_X(mF_t^u, F^+(v)) + \rho\cdot v}{\sqrt{1-\rho^2}}\right)\cdot \frac{n(v)}{\sigma_F\cdot\sqrt{T-t}\cdot\sqrt{1-\rho^2}}\cdot dv$$

$$\begin{aligned}\frac{\partial \mathbb{X}(>F^+,<)}{\partial G_t} = &-\frac{n(d_X(m_G G_t, k))}{\sigma_G\cdot\sqrt{T-t}} N\left(\frac{d_X(mF_T^u, F^+(k)) + \rho\cdot d_X(m_G G_t, k)}{\sqrt{1-\rho^2}}\right)\\ &+\int_{-\infty}^{-d_X(m_G G_t, k)} n(v)\cdot n\left(\frac{d_X(mF_T^u, F^+(v)) + \rho\cdot v}{\sqrt{1-\rho^2}}\right)\cdot\frac{dd(v)}{\sqrt{1-\rho^2}}\cdot dv\end{aligned}$$

with $dd(v) = \frac{\partial d_X(mF_T^u, F^+(v))}{\partial G_t} = -\frac{(F^+(v)-k)^2}{2\cdot F_T^+(v)\cdot k^{1/2}\cdot {G_t}^{3/2}\cdot\sigma_F\cdot\tau^{1/2}}$

$\mathbb{X}(<,<)$ is derived in a very close fashion, and the weight strategy follows.[10] Finally,

$$S_T = G_T \cdot \mathbb{1}_{m^* \cdot F_T^u > F^+(G_T)} + (G_T - k) \cdot \mathbb{1}_{m_G \cdot G_T > k \text{ and } m^* \cdot F_T^u \leqslant F^+(k)}$$

so the shares are priced as:

$$S_t = G_t \cdot \mathbb{Y}(>F^+,<) + G_t \cdot \mathbb{Y}(>F^+,>) + (1-m_G) \cdot G_t \cdot \mathbb{Y}(<,<) + G_t \cdot \mathbb{Y}(>F^+,<) - k \cdot \mathbb{L}(>F^+,<)$$

5 Empirical analysis

5.1 Visual presentation

As shown by Petajisto (2013), the active risk taking from fund managers has experienced an important downwards trend beyond the business cycle. Figure 2 and 3 show the mean active share and tracking error over time, and 4 show a boxplot of the distribution of the tracking error of non-indexed funds.

In these graphs, at each date, the *left panel* shows funds which have peers (other funds managed by the same asset management company, according to CRSP descriptive data), while the *right panel* shows all funds.

Out of a total of 3333 funds, 2983 funds that have peers (at at least one date), of 2584 are non-indexed funds. This leaves us with 61803 observation (funds * dates), and 50565 for non-indexed funds.[11]

[10] Note first that $\frac{\partial F^*}{\partial F^u} = \frac{\partial F_T^*}{\partial F^u}$, since $\frac{\partial F^*}{\partial F^u} = \frac{\partial F^*}{\partial F_T^*} \cdot \frac{\partial F_T^*}{\partial F^u} \cdot \frac{\partial F^u}{\partial F^u}$; then the weight strategy is $\pi_G = \frac{\partial F_T^*}{\partial G} \cdot \frac{G_t}{\partial F_t^*}$, and $\pi_{PSP}^* = \frac{\partial F_T^*}{\partial F^u} \cdot \frac{F_t^u}{F_T^*} \cdot \frac{\partial F^u}{\partial PSP} \cdot \pi_{PSP}$ (since $\frac{F^u}{F^*} = \frac{F^u}{F_T^*}$ and $\frac{\partial F^u}{\partial PSP} = \frac{1}{\gamma} \frac{\mu_{PSP}}{\sigma_{PSP}^2} = \frac{rr_{PSP}}{\gamma}$).

[11] The actual number of data point in the regression table is slightly lower, because the TE and AS is not always available. The actual number of observation is not systematically reported in tables 2 - 3, but it equal to one plus the number of observations plus the number of variables in the line *degrees of freed.*

Figure 2: Mean Active Share

Figure 3: Tracking error

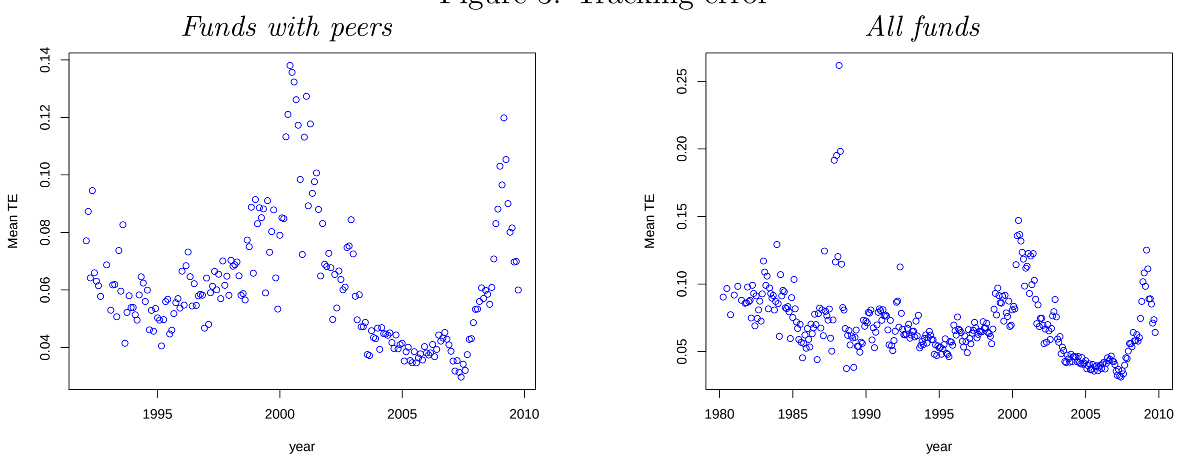
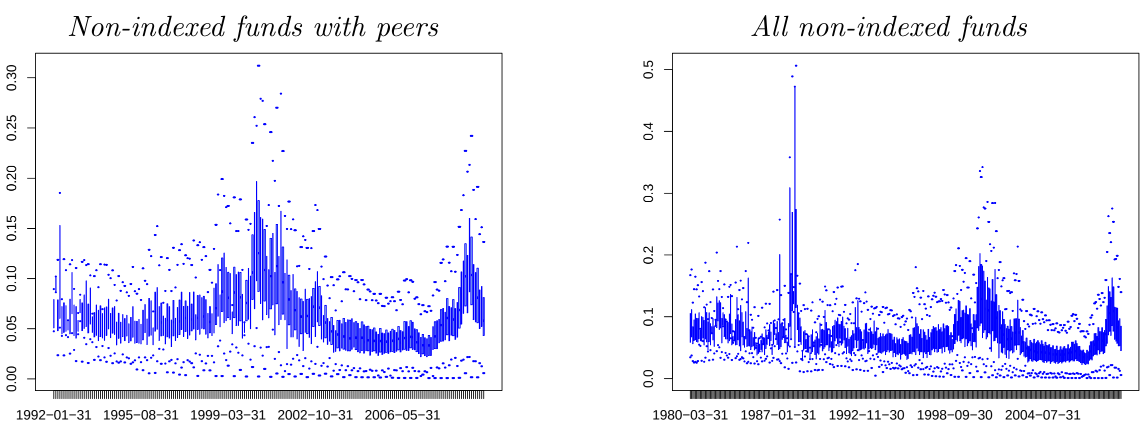


Figure 4: Mean Tracking Error



The whiskers are omitted and only represented by their staples or end points. The outliers are also omitted for clarity. The box width is a function of the number of funds reported at each date - they are thicker towards the end of the sample.

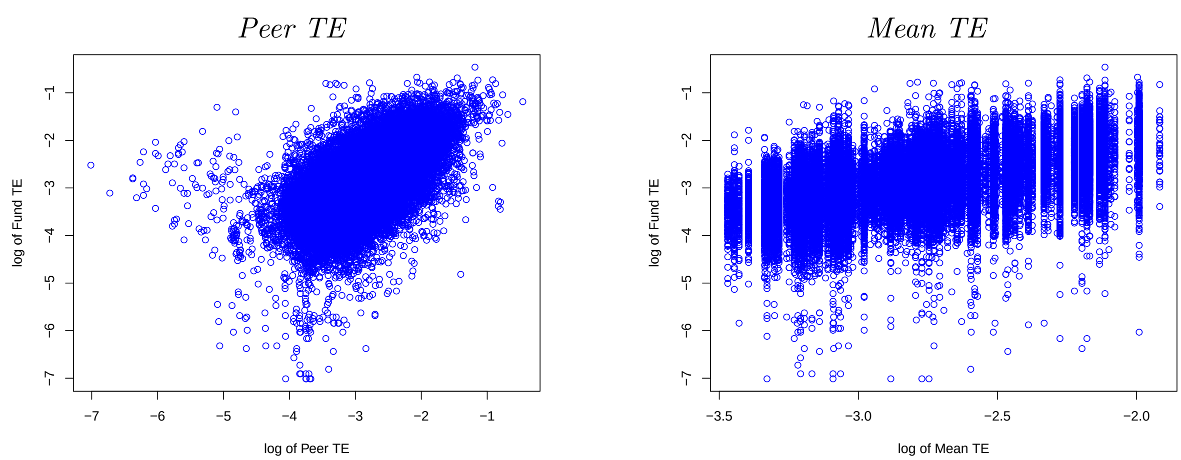
5.2 Preliminary analysis

Measuring the influence of the financial fragility of the firm requires additional data collection, and will restrict the number of asset management companies.

We thus seek preliminary evidence of the influence of firm conditions by assessing whether, before taking fund-specific structural information into account, the fund tracking error, hereafter AS (resp. active share, hereafter AS) clusters within fund management companies. To do so, we regress the fund TE against two variables, the mean of the Peer TE (resp Peer AS) taken as the mean of the TE of other funds managed by the same company at the same date, and the global Mean TE (resp global Mean AS) taken as the mean of the TE of all[12] funds reporting at that date.

Figure 5 gives an illustration of the relationship between the *Fund TE* and the *Peer TE* (left panel), and between the *Fund TE* and the *Mean TE* (right panel). The plot are in log for purely visual reasons (for the regressions, not qualitative difference arises when taking logs).

Figure 5: Non-indexed funds: Fund Tracking Error against Mean TE and Peer TE



The Peer TE has better explanatory power, see numerical data.

As shown by the x-axis on the left panel of 5, the *Peer TE* is (by construction) more volatile than the *Mean TE*. That it has a bigger explanatory power is thus a meaningful

[12] All funds within the universe of those having peers, including peer funds, but excluding the reference fund

indication of the influence of the manager's behaviour by company conditions.

Table 1: Summary statistics for the 61,800 observations of date/funds

Statistic	Mean	St. Dev.
Active Share (%)	74	24
Tracking error (%)	6.3	4.4
Number of peers	7.7	7.1
Index funds (%)	6.9	25

For the active share [AS] (resp. the tracking error [TE]), the main explanatory variable is the mean AS (resp., TE) of funds from peers, *i.e.*, , from the other funds managed by the same family.

Tables 2 and 3 focus on non-indexed funds,[13] but the results are qualitatively similar for all funds.

For the TE and AS alike, the most significant model is that including only the attitude of peers.

It is possible that TE and AS play a very different role in the risk taking of funds. Following Petajisto (2013), interpreting TE as systematic exposure to the MVE, and AS as a skill exposure (orthogonal to the MVE), is a possible explanation or the better explanatory power of both market conditions and peers' attitude.

After all, the relative exposure of all managers without skills is similar (it is an exposure to the MVE, in theory uniquely defined, in practice very likely well correlated to main market indices or *market risk*). Managers with such exposure are thus likely to overperform or underperform at the same time, their incentives and risk taking would thus tend to react similarly to important market changes.

Detailed analysis is on-going and will follow in updated versions of this paper.

[13] For all funds, the dummy for index is highly significant, and explains a large fraction of the variance, especially for AS, leading to (artificially) higher R^2.

Table 2: Linear models for TE of non-indexed funds

	Dep. var.: TE, non-indexed funds w. peers		
	(1)	(2)	(3)
Mean TE	1.019***		0.498***
	(0.006)		(0.007)
Peer TE		0.760***	0.529***
		(0.004)	(0.005)
Constant	0.0001	0.017***	0.00000
	(0.0004)	(0.0003)	(0.0004)
Adjusted R^2	0.333	0.393	0.437
Residual Std. Error	0.035	0.034	0.032
F Statistic	28.7e3***	37.3e3***	22.3e3***
Degrees freed.	(1; 57519)	(1; 57511)	(2; 57510)

The best model by far uses *Peer TE* alone: the F-statistic in (2) is greater than (1) with *Mean TE*, and (3) *Mean TE+Peer TE* degrades the F-statistic. Within non-indexed funds, the *Peer TE* explains 40% of the cross-section of the variance, and this is the most significant model.

Table 3: Linear models for AS of non-indexed funds

	Dep. var.: AS, non-indexed funds, peers		
	(1)	(2)	(3)
Mean AS	0.778***		0.437***
	(0.018)		(0.017)
Peer AS		0.395***	0.365***
		(0.005)	(0.005)
Constant	0.210***	0.489***	0.187***
	(0.013)	(0.003)	(0.012)
Adjusted R^2	0.033	0.117	0.126
Residual Std. Error	0.161	0.154	0.153
F Statistic	2e3***	7.6e3***	4.2e3***
Degrees freed.	(1; 57522)	(1; 57522)	(2; 57521)

Note: Observations: 57,524 — *** denotes $p < 0.01$

6 Conclusion

This paper details the optimal strategy pursued by a non-contractible agent endowed with an option-based compensation plus a salary guaranteed by the principal, when the guarantee is risky.

The agent takes into account the changes in the principal's wealth to define the amount of risk it can take; furthermore, the agent hedges against changes in the principal's wealth; then an increase in the principal's wealth diminishes the value of the funds managed by the agent.

When the principal's wealth falls, the agents takes less risk, reverting close to the benchmark, but maintaining a hedge position against the risk of further deterioration of the principal's wealth.

In a fund management context, one would expect that during times of crisis, the manager who has underperformed closes all her active positions (reducing tracking error to very low levels), while remaining underweighted in the stock of the financial company that employs him when it is traded.

Preliminary empirical analysis shows the managers' unconditional risk-taking behaviour (without taking her relative performance into account) is primarily influenced by firm-level conditions, more than by aggregate ones, and we expect to show that the main driver is precisely the financial fragility of the principal.

Other implications are expected to be testable with time. It would thus be expected that fund managers outperform their benchmark during recessions (provided that their principal, the asset management company, is too affected), but while underperforming at the start of expansions/recovery periods, only taking incentives into account. It is also expected that the greatest over and underperformance are realised during 'booms', when the principal has solid wealth.

It is also expected that hedging demands imply the manager being underweighted the principal's assets when she underperforms during economic downturns, which potentially biases downwards measures of performance (the sole active position of the manager may be to underweight the principal's shares, which is thus a rather undiversified position.)

In theory, such a structural model could help measuring manager's alpha and skills, suppressing a potential source of bias in previous structural models, the dependence of the manager's risk aversion to the financial fragility of her sponsor. However, to bring the model to the data, one needs both details fund holdings and measure of financial fragility, so, this question will likely be left for further research.

A The Economy and the Set-Up of the Model

A.1 Definitions and the Economy

The assumptions and notations are generic notations for either benchmark, numeraire or liabilities, but take into account the principal's riskiness and the interaction of the accounts of principal and agent.

We consider a continuous-time, finite-horizon $[0,T]$ economy where prices are exogenous to the agent. Uncertainty is represented by a filtered probability space $(\Omega, \mathfrak{F}, \{\mathfrak{F}_t\}, \mathbb{P})$, on which an n-dimensional Brownian motion W is defined. All stochastic processes are assumed to be adapted to $(\mathfrak{F}_t, t \in [0,T])$, the augmented filtration generated by W. The market is complete and all stated (in)equalities involving random variables hold $\mathbb{P}$-*almost surely*.

The vector of traded risky assets P follows the process: $dP_t = diag(P_t) \cdot (\mu_t \cdot dt + \sigma_t \cdot dW)$ where the vector μ_t represents the risk premium and the matrix σ_t stacks the volatility of the assets. The agent's benchmark L_t has volatility σ_{Lt}. We denote θ the risk premium process, $\theta_t = \sigma_t^{-1} \cdot (\mu_t - r_t)$.

The stochastic discount factor, M, has diffusion $dM_t = -M_t \cdot (\theta_t' \cdot dW + r_t \cdot dt)$ and value $M_t = \exp(\int_0^t -\theta_t' dW - \int_0^t [r_t + \frac{\|\theta_t\|^2}{2} dt])$

The martingale measure $\mathbb{Q}$ is defined by $\zeta_t = M_t \cdot e^{\int_0^t (r_t \cdot dt)}$ with respect to the historical probability measure $\mathbb{P}$. As the utility is on the funding ratio, *i.e.*, on a measure of wealth where the liability serves as a numeraire, we use, by default, the corresponding liability forward martingale measure $\mathbb{Q}_L$. $F_T = \frac{A_t}{L_t}$, the funding ratio at time $0 \leqslant t \leqslant T$, is a martingale under $\mathbb{Q}_L$, so that in all equations, the budget constraint on A_T is written instead as a budget constraint on A_T under $\mathbb{Q}_L$, without discounting.

The principal has a net asset value Y denoted by $G = \frac{Y}{L}$ in the numeraire of the liability. G has volatility $\sigma_G = \sigma_Y - \sigma_L$. The share value S will be determined endogenously.

For notational economy, we denote $F_T^u = (M_T \cdot L_T)^{-\frac{1}{\gamma}}$ the optimal asset to benchmark ratio of an unconstrained CRRA investor with relative risk aversion γ. The manager will in the main choose her allocation to F_t^u (or $A_t^u = F_t^u \cdot L_t$) and her allocation to the replicating portfolio of G.[14] We also suppress the time subscript on volatilities.[15]

m, the participation to A^u_T, is a positive number chosen so that the budget constraint is satisfied. H_T represents the 'total' terminal funding ratio after receiving in T recovery cp_T.

π denotes the portfolio weights. $[a \vee b] = max(a,b)$ and $[a \wedge b] = min(a,b)$.

In closed-form solutions, n denotes the standard normal density $(n(z) = \frac{1}{2\pi}e^{-\frac{z^2}{2}})$, N denotes the normal cumulative distribution, and $B(x,y,\rho)$ is the cumulative Bivariate normal where the marginals x and y are standard normals and their correlation is ρ.

B Derivation of the Optimal Payoffs

Remark B.1. A^u driver of the strategy:

First we solve the **unconstrained** static program: $\max_{\pi} E_0 \left[\frac{(A_T/L_T)^{1-\gamma}}{1-\gamma}\right]$

$$s.t.\ A_0 = \mathbb{E}_0^{\mathbb{Q}}[\frac{A_T}{e^{rT}}] = \mathbb{E}_0[M_T \cdot A_T] = L_0 \cdot \mathbb{E}_0^{\mathbb{Q}_L}[\frac{A_T}{L_T}] \Longleftrightarrow \mathbb{E}_0^{\mathbb{Q}_L}(A_T) = A_0$$

This solution thus uses the martingale approach of Cox and Huang (1989).[16] Given the states of the world, the first-order condition reads $\frac{1}{L_T}\left(A^u_T/L_T\right)^{-\gamma} = \upsilon_0 \cdot M_T$. So, $F^u_T = \upsilon_0{}^{-1/\gamma} \cdot M_T{}^{-\frac{1}{\gamma}} \cdot L_T^{1-\frac{1}{\gamma}}$, and $F^u_T = \upsilon_0{}^{-\frac{1}{\gamma}} \cdot (L_T M_T)^{-\frac{1}{\gamma}}$

Similar in spirit to El Karoui et al. (2005), we now use the unconstrained A^u as a driver of the strategy, and without loss of generality we set here $\upsilon_0 = 1$.

Theorem B.2. *Generalised Risk-Shifting Portfolio Insurance*

The optimal $(X^*_T/L_T, F^*_T)$ *are:*

$$X^*_T/L_T = \begin{cases} m^* \cdot F^u_T & \text{if} \quad m^* \cdot F^u_T > F^+(G_T) \\ k & \text{if} \quad m^* \cdot F^u_T \leqslant F^+(G_T) \end{cases}$$

[14] We thus rely on an 'asset-space' representation where A^u is is used as a univariate driver of the allocation, when the *s.d.f.* representation involves both M and the liability L.

[15] Thus σ_{F^u} represents the instantaneous $\sigma_{F^u,t}$ for the weight process and $\sigma^2_{F^u}$ stands for $\int_\tau^T \sigma^2_{F^u,\tau} d\tau$ in option pricing formulas.

[16] In complete markets, the duality or martingale approach used is equivalent to solving the Hamilton-Jacobi-Bellman partial differential equation, but reliance on a static optimisation of the optimal terminal payoff is simpler.

and

$$F_T^* = \begin{cases} 1 + (m^* \cdot F_T^u - k)/\alpha & \text{if} \quad m^* \cdot F_T^u > F^+(G_T) \\ F^-(G_T) = [k - m_G/\beta G_T]^+ & \text{if} \quad m^* \cdot F_T^u \leqslant F^+(G_T) \end{cases}$$

where $F^-(G_T) = [k - m_G/\beta G_T]^+$ *denotes the low reference funding ratio, and* $F^+(G_T)$ *the high threshold solves* $\frac{(k+\alpha\cdot(F^+-1))^{1-\gamma}}{1-\gamma} - \frac{k^{1-\gamma}}{1-\gamma} = (F^+ - F^-)\cdot\alpha\cdot(k+\alpha\cdot(F^+-1)^{-\gamma}$

Remark B.3. Proof *à la* Carpenter (2000).

It is possible to extend directly the proof of Carpenter (2000). Indeed, in the static optimisation problem at time-T, conditional to G_T, the non-convex problem is similar, but with the minimum value $F-(G_T) = [k - m_G/\beta G_T]^+$ to avoid zero terminal wealth, and an upper value $F^+(G_T)$ that depends on $F^-(G_T)$ thus on G_T. We propose below a more detailed proof below.

Proof. of theorem B.2. The duality approach can be used in non-convex maximisation, see Basak and Shapiro (2001). Carpenter (2000) 'convexifies' the utility function, but by standard non-convexity arguments we can also directly show that the agent's terminal funding ratio F_T does not take any step in the stochastic non-convex region $]F^-, F^+[$, so that it is only when $m \cdot F_T^u > F^+$ that the standard convex solution is recovered. Below this threshold, the manager always targets precisely the unique F^-.

C Graphical Illustration of the Strategy

The benchmark L is constant, and the problem normalised with $\alpha = \beta = 1$; the multiplier $m^* = 1.8$ with a guarantee $K = 1$ corresponds to a value $k' = 1.54$ at time 0 of the compensation package (which can be interpreted as an option package worth 50% of the value of the fixed salary). The agent has risk aversion $\gamma = 2$; interest rates are constant and $\sigma_{F^u} \equiv \sigma_{A^u} = 0.12$ (constant); its drift is $\mu_{F^u} = 2\%$; $\sigma_G = 0.25$ and $\rho = .3$

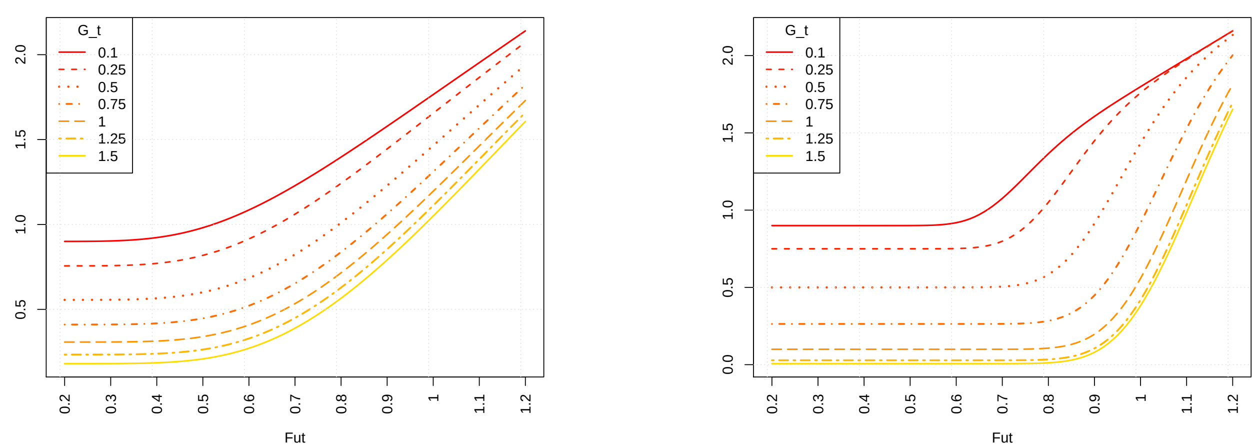
The following set of graphs show for $\tau = T - t$ at either 5 or 1 year the deltas or π_{PSP} the weights strategy with respect to the speculative portfolio, π_G with respect to either the principal's net asset value, and π_S the weight allocated to the principal's shares.

Note that up to the point where the principal's shares S are valued, and used as a hedging instrument, m_G plays no role, thus the first set of graphs can be thought as plotted with $m_G = 1$, or the G axis representing $m_G \cdot G_t$ and derivatives being taken *w.r.t.* $m_G \cdot G_t$.

C.1 Value of the Funds Managed by the Agent

The value of the funds F_t^* is an increasing function of F^u. The value of the principal support increases in G, so that F_t^* decreases in G_t.

Figure 6: Relative value of the funds, 5 Year horizon (left) and 1-Year (right).

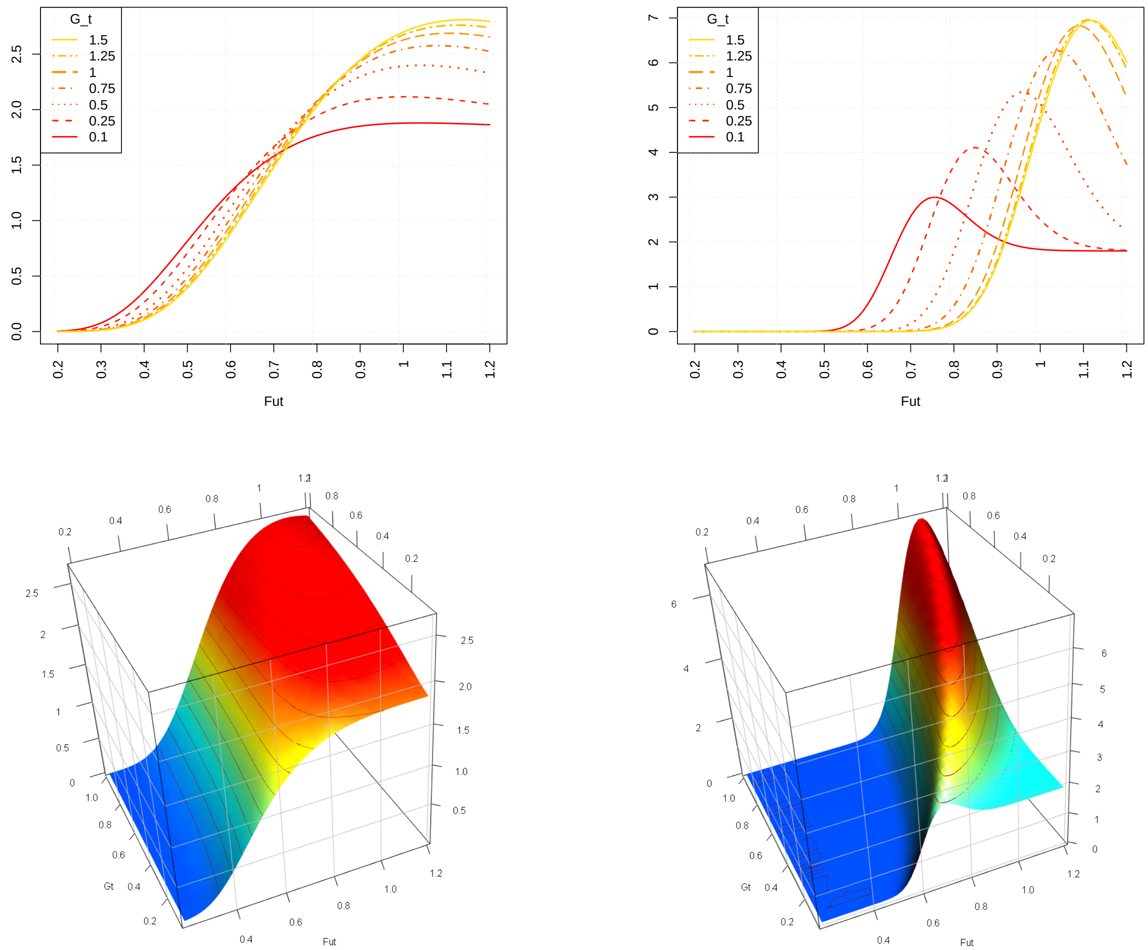


The kink when F^u is between .8 and 1 is magnified when τ is small [1Y]; F_T^* decreases in G_T: a higher G_T decreases non-only the value of the put option on G_T, but also that of the *up* option, because a higher G_T increases $F^+(G_T)$ the strike of *up*.)

C.2 Optimal Exposure to the PSP when G is Traded

F_T^* takes no value in $]\![k - m_G \cdot G_T]^+, F^+(G_T)[$, which creates 'local leverage'. The width of the non-convex region $[[k - m_G \cdot G_T]^+, F^+(G_T)]$ increases with G, which in turn means that the local leverage component increases with G (until $m_G \cdot G_T > K$) (a bigger bet is needed to achieve bigger outcomes). The hump increases when $t \uparrow T$.

Figure 7: $\frac{\partial F^*_{T-\tau}}{\partial F^u_t}$, with $\tau = 5Year$ (left) and $\tau = 1Year$ (right)



These figures show the delta of the strategy wrt F_t^u (shape of the risk-taking strategy) when $\tau = (1,5)$.
F_t^* has a positive delta to F_t^u because a rise in F_t^u always raises the probability of being into the high-value region, and because it raises the value of the exposure to F_t^u.

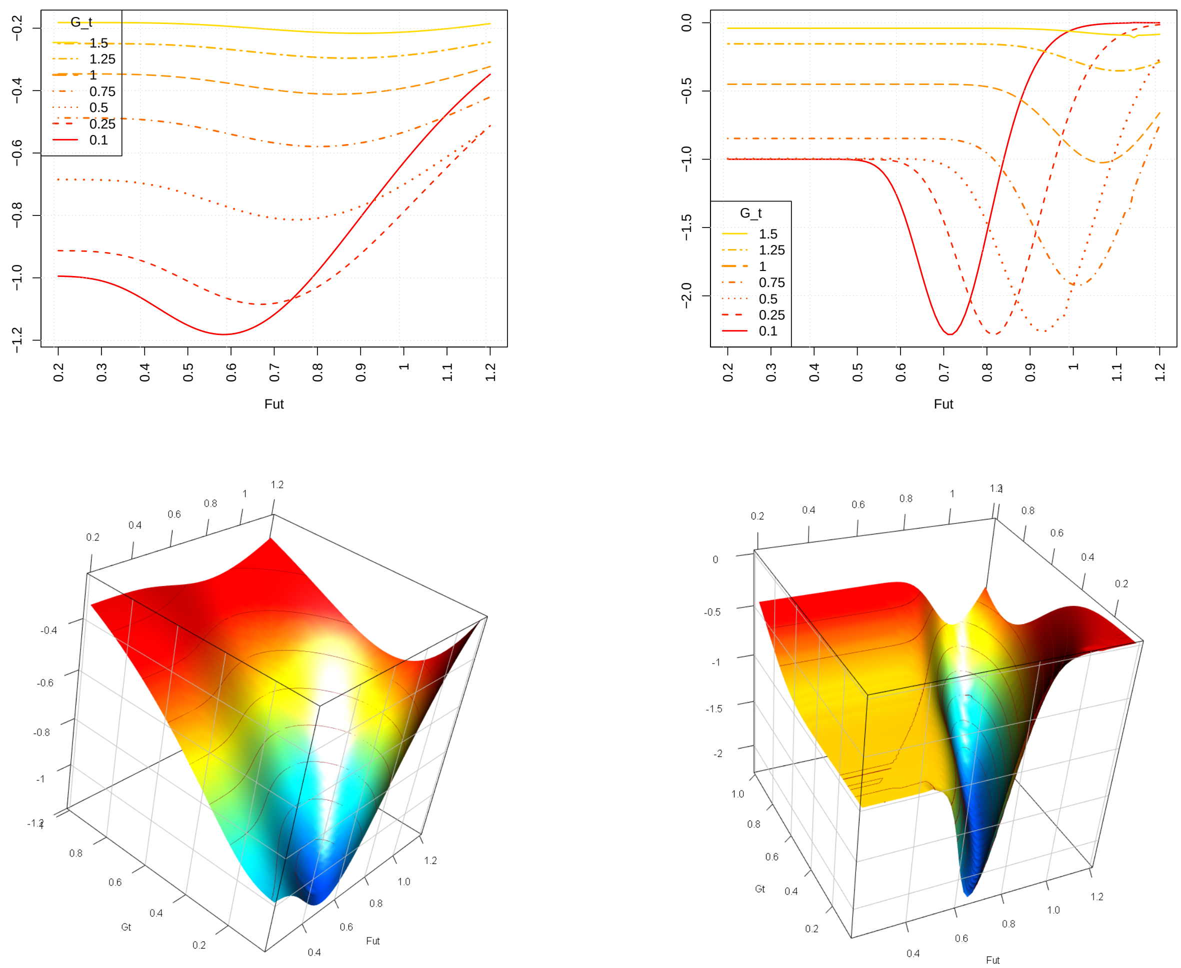
When τ is small, the delta *w.r.t.* G is very humped-shape. The agent builds 'local leverage' to bet himself out the non-convex region $[F^-, F^+]$. The magnitude of the bet increases linearly in $\frac{1}{\sqrt{T-t}}$, and increases in G_T up to the point where $m_G \cdot G_T = k$.

C.3 Hedging of Principal's Risk when G is Traded

F_t^* has a negative delta to G_t because the value in case of failure is a put option on G_T, and because a higher G diminishes the probability of success of up_T ($F^+(G_T)$ is an increasing function of G_T).

In addition, since F^* is leveraged, the delta of F^* *w.r.t.* G_t also is. It can thus be lower than $-m_G$.

Figure 8: $\frac{\partial F^*_{T-\tau}}{\partial G_{T-\tau}}$, with $\tau = 5Year$ (left) and $\tau = 1Year$ (right)



A distinctive feature in figure 8 is that the sensitivity of the value of the funds to G, while always negative, is not a monotonic function of F. In particular, the local leverage component means that this sensitivity is magnified at given 'intermediary' values (near the forward moneyness). $\frac{\partial F^*_{T-\tau}}{\partial G_{T-\tau}}$ is always negative, and it can reach values that are a multiple of $-m_G$.

D Generalisations

The idea of this section is three-fold:

1) clarify why this use of a numeraire L simplifies the derivation.

2) state that many variations of the form of the payoff can be considered (in fact, conditionally on the random benchmark, we will recover the problem we have here detailed, and in particular, conditioning again on Y_T, the problem will be of known form.

3) state that this can be extended to any concave utility. Of course, pricing and hedging will be more demanding then.

D.1 Why the simplifications

We note that by denoting $F = \frac{A}{L}$, we have in our problem $F^- = (1 - K/\beta)$ independent of L_T, and, provided the CRRA utility is on the terminal funding ratio F_T, we also have that F^+ will be independent of L_T. Such a simplification permits notational economy, since the expectations will not need to be conditioned on L_T (it is then also computationally simpler), but does not modify the essence of the problem, see also Appendix D.

Here, thus, we have only one level of integration. If as in Koijen (2014) and Buraschi et al. (2014) we have a function involving various assets jointly, then one level of integration is needed to compute F^u and σ_{F^u}, and another for the pricing of options on this portfolios, involving $F^u_T|G_T$. Note that in this case, when needed, the additional complexity stems principally from $G_T|F^u_T$.

In general, G_T can have any form of correlation with A_T, but it will of course be convenient in the application to consider a constant correlation (small manager even with non-linear performance fees on AM that also benefit the principal in a non-linear way; large entity where assets, though segregated, can be thought as belonging to the principal, who diversifies).

D.2 Fixed remuneration

Consider the problem $X_T = \alpha \cdot (A_T - L_T)^+ + k \cdot \mathbb{1}_{\beta \cdot [L_T - A_T]^+ \leq m_G \cdot Y_T}$

By conditioning on L_T, one sees clearly that the optimal payoff in the success region will be $up_T = (m \cdot A_T^u - k/\alpha + L_T) \cdot 1_{m^* \cdot A_T^u \geqslant A^+(G_T)}$ and $put_T = [L_T - \frac{m_G}{\beta} Y_T]^+ \cdot 1_{m^* \cdot A_T^u < A^+(Y_T)}$.

This problem will generally have dependence on L_T in addition to the one we study on Y_T (or G_T). The very form of A^+ depends on the utility function considered (see note above).

D.3 Generalisation of the MRS Theorem to any Differentiable Concave Utility

We note that the Maximum Risk Shifting Theorem directly extends to any concave function $U(A,L)$. Denoting I_L the inverse function of $\frac{\partial U}{\partial A}$, we have, for a λ such that the budget constraint holds, and with M_T the stochastic discount factor

$$A_T^* = [L_T - \frac{m_G}{\beta} G_T]^+ \cdot 1_{I_L(\lambda M_T) < A^+(G_T)} + \frac{I_L(\lambda M_T)}{L_T} \cdot \mathbb{1}_{I_L(\lambda M_T) < A^+(G_T)}$$

The put part has the same form in the exercise region regardless of the utility function (which however determines the exercise region), so the same singularity holds with any utility function when using principal shares as hedging instruments.

D.4 Generalisation of the MRS Theorem with a Benchmark plus a Utility Numeraire

In the above, we have considered that the numeraire L of the utility function also serves as a benchmark for compensation. This is arguably the case for an institutional investor who manages assets relative to liabilities such as nominal or real annuities. In other contexts, for instance, in fund management, the benchmark may be fund-specific (e.g. an equity index) while the numeraire could be a more standard real bond.

When the agent's performance is assessed relative to a benchmark that is not relevant as a numeraire to her utility function, for instance, we denote B_T the ratio of the benchmark to the relevant numeraire L_T. While a variety of compensation packages are possible, the remuneration H_T can be simply defined as:[17]

$$H_T = \begin{cases} k + (A_T - B_t) & \text{if} \quad (A_T \geq B_T) \\ k - [B_T - A_T - m_G \cdot G_T]^+ & \text{if} \quad (A_T < B_T) \end{cases} \tag{10}$$

In problem (10) the lower threshold A^-, relative to B_T, is: $A^-(G_T, B_T) = [B_T - m_G \cdot G_T]^+$; above A^+ (where $[A^-, A^+]$ is tangential to the utility curve at A^+) then agent targets a total 'real' wealth that is proportional to A^u. Thus as $H_T|(A_T > B_T) = m \cdot A_T^u$, we have $A_T|(A_T > A^+) = B_T + m \cdot A_T^u - k$, and we note that $A^+ \geq max(B_T, k)$

E Review of Related Remuneration Functions

We call the *seminal incentive problem* that of Carpenter (2000), where the agent has an option-based compensation plus a fixed, guaranteed payoff: the manager's manages assets denoted A, and her terminal wealth X_T is:

$$X_T = \alpha \cdot [A_T - L_T]^+ + k \tag{11}$$

, where L is the benchmark.[18]

Assuming that the agent has power utility on her total terminal wealth X_T: $U(X_T) = \frac{X_T^{(1-\gamma)}}{1-\gamma}$, the manager will ensure that her total wealth is either k or $m^* \cdot X_T^u$, where M is the state-price density and $A_T^u = M_T^{-1/\gamma}$ is the result of a fixed-mix investment strategy between the risk-free rate and the mean-variance efficient (hereafter MVE) portfolio M_T^{-1}. In turn,

[17] When the benchmark was the numeraire ($L_T = B_T$), it was convenient to simplify the problem, rescaling the AUMs and the benchmark to eliminate α, further taking the convention for expository purposes that the strike of the option is at k.

When the benchmark and the numeraire differ, such simplifications are not possible without loss of generality. Indeed, even when the manager beats her benchmark, it is still possible that the fees do not cover the salary. There are thus more possible variations to the problem.

In (10) we only rescale the asset and benchmark values so that $\alpha = 1$.

[18] We will reserve the letter B for the cumulative bivariate' normal distribution.

this means that she will target a portfolio performance of

$$A_T^* = \begin{cases} 0 & if \quad m^* \cdot A_T^u \leq \hat{A} \\ \frac{m \cdot A_T^u - k}{\alpha} + L_T & if \quad m \cdot A_T^u > \hat{A} \end{cases} \tag{12}$$

The manager invests in the MVE portfolio to outperform her benchmark, which results in particular in leveraged positions on the MVE portfolio when it is the benchmark. When the state price of risk is very low and the MVE performs, the portfolio will tend to $m/\alpha \cdot A_T^u + L_T$, and if L is the MVE, the portfolio weights will asymptotically be those of the MVE if the risk-aversion parameter $\gamma > 1$,[19] otherwise a proportion $\frac{1}{\gamma}$ thereof, *i.e.*, the Merton constant.[20]

In this *seminal incentive problem*, the optimal risk allocation involves a leverage component that is a monotonic function of the moneyness of the option. As the agent (the manager or portfolio manager depending on the context) 'shuns payoff that are likely to be near the money' (Carpenter, 2000, p231), the leverage increases when the value of the assets falls, and becomes infinite when the asset value approaches zero.

Contracts that are designed ex-ante to offer unlimited, unconditional guarantees in all states of the world, thus yielding unlimited leverage and zero value of the assets are arguably not universally fit, and the recent literature has included some limitation to the under-performance that the principal is willing to tolerate from the asset manager; for hedge funds, the 'capital structure fragility induced by the possibility of investors withdrawing capital (and prime brokers forcing deleveraging)' (Buraschi et al., 2014, p2820). A slightly simplified[21] version of the manager's compensation in (Buraschi et al., 2014, p2825) is:

$$X_T = \alpha \cdot (A_T - L_T) + k \cdot L_T \cdot \mathbb{1}_{\beta \cdot [L_T - A_T]^+ \geq K \cdot L_T} \tag{13}$$

[19]It will also be the case if L is a riskless benchmark.

[20]The Merton constant are the weights of A^u in case the agent has CRRA utility on the terminal wealth. Note that when the benchmark has exposure to other factor than the MVE, the portfolio weights in Carpenter (2000) being asymptotically determined by the dominating portfolio, they usually depend on the asymptotic value of the ratio of the MVE to the benchmark. For this reason, we will sometimes rely on a notional F^u and subsequent change of numeraire techniques.

[21]The hedge fund manager-entrepreneur of Buraschi et al. (2014) receives a variable compensation equal to the management fees on indexed to the value of the fund A^*, without taking performance driven flows into account.

Here, k is the fixed remuneration of the manager, while $\beta \cdot [L_T - A_T]$ represents the loss that results from under-performance, which needs to be no larger than the fixed salary $K \cdot L_T$ (or that any predetermined value only related to A and L).

In this case, the manager will ensure that $\beta \cdot [L_T - A_T]^+$ is never bigger than $k \cdot L_T$, which defines $A^-(L_T) = (1 - k/\beta)L_T$. The utility of $X(A)$ is non-convex between A^- and $A^+(L_T)$ defines such that $\frac{U(X(A^+)) - U(K)}{A^+ - A^-} = (dU)(A^+)$ where (dU) is the derivative of the utility function.

Note again that the non-convex region is identical over a variety of configurations.[22]

Unless (as possibly in hedge funds) the manager is truly an entrepreneur, she receives a wage compensation, which in most relevant situations entails some risk, for instance job loss if the principal's financial situation[23] deteriorates strongly while the manager under-performs, an aspect which can be modelled as a default risk of the principal when the agent's losses are high compared to the principal's wealth.

So, the agent we study has *limited liability* up to a certain under-performance that depends on the stochastic wealth of the principal.

Maintaining the salary is thus a put option provided by the principal [the employer], and it is useful to rewrite the problem as the one we study:

$$\begin{aligned} X_T &= \alpha \cdot (A_T - L_T)^+ + K \cdot L_T \cdot \mathbb{1}_{\beta \cdot [L_T - A_T]^+ \leq m_G \cdot Y_T} \\ & with \\ U(X_T) &= \frac{(X_T/L_T)^{(1-\gamma)}}{1-\gamma} \end{aligned} \tag{14}$$

[22]For performance as 'risk-shifting' incentives, the wage can be indexed to A as long as incentives provide an additional indexation to A; for risk-control incentives, the fund manager may have a deep des-utility of wealth below certain level, face job risk or the risk of a loss in total wage.

[23]It could be chosen to model the financial fragility of a third party*e.g.,* that of the prime broker. This represents the case when G, the net asset value of the provider of the guarantee, is directly traded.

References

Basak, S. and A. Shapiro (2001). Value-at-risk-based risk management: optimal policies and asset prices. *Review of Financial Studies 14*(2), 371–405.

Berkelaar, A., K. R. and T. Post (2004). Optimal portfolio choice under loss aversion. *The Review of Economics and Statistics 86*(4), 973–987.

Buraschi, A., R. Kosowski, and W. Sritrakul (2014). Incentives and endogenous risk taking: A structural view on hedge fund alphas. *The Journal of Finance*.

Carpenter, J. (2000). Does option compensation increase managerial risk appetite? *The Journal of Finance 55*(5), 2311–2331.

Cox, J. and C. Huang (1989). Optimal consumption and portfolio policies when asset prices follow a diffusion process *Journal of Economic Theory 49*(1), 33–83.

El Karoui, N., M. Jeanblanc, and V. Lacoste (2005). Optimal portfolio management with American capital guarantee *Journal of Economic Dynamics and Control 29*(3), 449–468.

Jensen, M. and W. Meckling, (1976). Theory of the firm: Managerial behavior, agency costs and ownership structure *Journal of Financial Economics 3*(4), 305–360.

Koijen, R. (2014). The cross-section of managerial ability, incentives, and risk preferences. *The Journal of Finance 69*(3), 1051–1098.

Petajisto, A. (2013). Active share and mutual fund performance. *Financial Analysts Journal 69*(4), 73–93.